Unsupervised learning of indoor localization based on received signal strength

Li Li¹, Wang Yang¹*, Md Zakirul Alam Bhuiyan² and Guojun Wang³,Y

¹ School of Information Science and Engineering, Central South University, Changsha, China
² Department of Computer and Information Sciences, Temple University, Philadelphia, PA, U.S.A.
³ School of Computer Science and Educational Software, Guangzhou University, Guangzhou, China

ABSTRACT

Most indoor wireless sensor network localization methods require costly site surveys to collect fingerprint information for later comparison. Moreover, due to the dynamic nature of fingerprint information in indoor wireless environments, the need for site surveys may be ongoing. In this work, indoor localization is addressed with an unsupervised learning algorithm. Our novel algorithm based on received signal strength combines the information conveyed by both range-based and range-free localization with state-of-art optimization techniques. A specially designed hierarchical Bayesian hidden Markov model coupled with a particle filter helps mitigate non-line-of-sight and multipath errors. This grid-based data sample process, derived from the theory of Dirichlet processes, simplifies the global optimization problem of unsupervised learning by employing a single initial hyper-parameter. Meanwhile, for obtaining accurate coordinates of mobile nodes, a unique semidefinite programming method is used to provide feedback to the radio propagation model. This feedback step can enable the grid-based algorithms not only to establish the coordinates of a mobile node, but also to optimize the accuracy iteratively. Theoretical and experimental analyses indicate that the proposed algorithm can achieve better localization accuracy than conventional range-based algorithms without adding computation cost. Copyright © 2016 John Wiley & Sons, Ltd.

KEYWORDS

semidefinite programming; hierarchical Bayesian hidden Markov model; unsupervised learning; wireless localization

*Correspondence

Wang Yang, School of Information Science and Engineering, Central South University, Changsha 410083, China.
E-mail: yangwang@csu.edu.cn

1. INTRODUCTION

Node localization in wireless sensor networks (WSNs) plays a key role in many indoor applications such as target tracking, robotic navigation, smart space, intrusion detection, and inventory management [1]. Time-of-arrival, time difference-of-arrival, angle-of-arrival, and received signal strength (RSS) are commonly used measurement units for indoor localization. Because WSNs are already equipped with the necessary RSS units for communication purposes, essentially no other measurement unit is required for RSS-based localization, and localization approaches can be layered over this existing capability with low computational cost. For this reason, we focus on RSS-based localization in the present work.

Localization in WSNs is typically divided into noncooperative versus cooperative approaches[2]. A noncooperative approach involves no communication between mobile nodes, but only between mobile nodes and reference nodes. Here, every mobile node must communicate with multiple reference nodes. However, a cooperative approach employs localization sensors that cooperate in a peer-to-peer manner to make measurements, and then form a map of the network. In this paper, we focused on noncooperative localization. Noncooperative localization in a WSN is similar to that of Wi-Fi localization. Additionally, the signal employed in a WSN is also similar to a Wi-Fi signal. In fact, numerous indoor wireless localization algorithms can be applied in both of these networks. In [3], Kleisouris et al. employed a fingerprinting algorithm in both an 802.11 (Wi-Fi) network as well as an 802.15.4 (ZigBee) network. These researchers found that multiple antennas improve the localization stability significantly. In [4], Pan et al. tested their semi-supervised learning algorithm based on a manifold with different hardwares such as an 802.11 wireless local area network (WLAN) and a WSN.

According to experimental findings, the distance between a transmitting node and a receiving node is related...
to the RSS value [5]. RSS-based localization utilizes this relationship to identify the location of a mobile node using RSS information with respect to multiple reference nodes. The method is generally classified into range-based and range-free localization approaches.

In a range-based localization algorithm, the RSS is converted to a distance according to a theoretical radio propagation model [6]. However, non-line-of-sight (NLOS) signal propagation and multipath propagation through indoor environments can introduce unpredictable deviations in the distance measurements derived from theoretical propagation models, and are therefore major challenges for indoor localization based on RSS values [5]. NLOS scenarios occur under conditions where no direct path exists between the transmitting and receiving nodes, and the interference of multipath signals can cause the received power at a given location to vary over time, resulting in the so-called fast fading phenomenon.

Numerous techniques have been reported recently to mitigate the errors arising from NLOS and multipath conditions [5,7,8]. Most of these can be viewed as falling into two main categories. (i) Assume that the measurement errors follow Gaussian distributions. Thus, global or regional optimization can be employed to reduce the location estimation errors. One popular approach based on semidefinite programming (SDP) has been utilized for error minimization [8]. (ii) Employ techniques based on pattern matching. Numerous localization algorithms use pattern-matching-based techniques [2]. In pattern-matching-based techniques, the RSS values obtained at different locations from a fixed set of reference nodes are used as fingerprint information. This technique is also referred to as a range-free localization approach. Because RSS patterns are directly mapped and translated into locations, measurement errors caused by NLOS and multipath conditions can theoretically be avoided. However, a significant investment of human labor is typically required to build a detailed map-database, and, because of changes in the fingerprint space of an indoor wireless environment, the investment is not necessarily a one-time expense. Fortunately, unsupervised learning methods do not associate any explicit category label with each fingerprint. Therefore, for reducing labor and time, our proposal is to use an unsupervised learning method.

In this paper, we propose a novel indoor wireless localization based on unsupervised learning (IWLUL) algorithm for acquiring the coordinates of indoor wireless sensor nodes. The objective of our algorithm is to maximize the expected localization accuracy by employing a method that seeks to effectively unify range-based and range-free approaches. Moreover, the selected optimization technique should provide for ease of use by employing fewer specific parameters [9]. In the IWLUL algorithm, we unify range-based and range-free localization using a hierarchical Bayesian hidden Markov model (HB-HMM), a particle filter (PF), and SDP. Generally, algorithms based on a hidden Markov model (HMM) depend on a set of initial parameters for global optimization [10]. However, our algorithm depends only upon a single hyper-parameter $\beta$, simplifying the global optimization problem of unsupervised learning. Experiments were conducted in an indoor environment using automatically collected data as a training database. The proposed unsupervised learning algorithm can be implemented for a wide range of devices and is an efficient and inexpensive solution.

The contributions of the algorithm proposed herein with respect to other localization methods are as follows:

1. The proposed method is low cost and outperforms conventional range-based algorithms by modeling and accommodating both range-based and range-free algorithms with an SDP method.
2. Provides improved localization accuracy by combining HB-HMM and PF.
3. Simplifies the global optimization problem of unsupervised learning using a single initial hyper-parameter.
4. The SDP method can enable the grid-based algorithm not only to obtain the coordinates of a mobile node, but also to optimize the accuracy iteratively.

The remainder of this paper is organized as follows: Section 2 provides an overview of related localization systems. The problem statement of indoor localization is introduced in Section 3. Subsequently, Section 4 describes the proposed algorithm. Section 5 discusses the indoor experiments. The paper closes with a conclusion in Section 6.

2. RELATED WORK

With the exception of range-based algorithms such as weighted least squares (WLS) [11] and multidimensional scaling [12], a number of localization techniques based on unsupervised learning have been developed for wireless technologies. For example, Wu et al. [13] designed an unsupervised localization approach denoted as wireless indoor localization based on k-means clustering and a logical floor plan mapping method for indoor wireless localization that eliminated the need for site surveys. Wireless indoor localization was applied to a building with a floor area greater than 1600 m$^2$, and a site survey was not required in its deployment phase. Clustering techniques are commonly employed for localization within wireless networks based on unsupervised learning. Wang et al. [14] proposed UnLoc, an unsupervised learning method for indoor localization. Using clustering, UnLoc established landmarks from a variety of sensors (e.g., magnetometer, sound, ambient light, etc.) to substantiate the localization estimates. Luís et al. [15] proposed a three-phase method for locating indoor mobile nodes employing a type of clustering approach based on a Kohonen network. A Kohonen network is also referred to as a self-organizing map (SOM), which is a type of artificial neural network that is trained using unsupervised learning. Experimental results indicated that the method was capable of correctly identifying the location of a mobile terminal with high
probability. Ferris et al. [7] proposed an unsupervised learning methodology for solving the Wi-Fi localization problem using a Gaussian process latent variable model. Carlo et al. [16] proposed a grid-based Bayesian approach based on a jump Markov system. In this approach, the HMM parameters were estimated by the well-known Baum–Welch algorithm. Li et al. [17] also proposed the HIWL algorithm based on an HMM that used prior knowledge to optimize the results. Pan et al. [18] proposed a semi-supervised learning approach based on manifold regularization for tracking mobile nodes in a WSN. Ahsishe [19] proposed a learning-based approach denoted as WiGEM, where the RSS is modeled as a Gaussian mixture model. The learning approach employed not only avoids labor-intensive training, but also makes the location estimates considerably robust with respect to the use of various devices and varying environments. Tsui et al. [20] proposed an unsupervised learning method for Wi-Fi localization under conditions of hardware variance, although their method was limited by local optimality owing to the use of expectation maximization.

However, these unsupervised approaches are not directly applicable to range-based localization techniques, and they cannot readily convert RSS data to distance. Jamma et al. [21] developed a method denoted as EasyLoc, where the RSS information of the reference nodes is frequently exchanged to derive an online and RSS-to-distance mapping. While EasyLoc does not require labor-intensive pre-deployment profiling operations, the trained database only contains fingerprints near reference nodes, and its performance diminishes in other regions.

### 3. PROBLEM STATEMENT

Consider a WSN deployed in an indoor environment, where the nodes of devices with fixed positions are employed as $M$ reference nodes, and the locations of mobile nodes must be determined by a localization estimator. Location information derives solely from the RSS between mobile nodes and reference nodes. Specifically, a vector representative of the signal strengths received by a mobile node from different reference nodes, namely, the fingerprint of location $i$, is defined as

$$S_i = \{s_{i,1}, s_{i,2}, ..., s_{i,M}\} \quad i = 1, 2, 3, ..., N$$

where $s_{i,j}$ denotes the signal strength received from the $j$th reference node. $S_i$ refers to the $i$th observation in a time sequence, and $N$ represents the total number of observations. If $S_i$ is given, then our goal is not only to obtain the grid representative of the mobile node's location, but also to obtain the coordinates of the mobile node.

Range-based localization is highly dependent on the radio propagation model employed. A popular model employed under NLOS and multipath conditions relates the RSS values to the distance as follows:

$$\text{RSS} = L_0 + 10 \gamma_{\text{NLOS}} \log_{10} d + S_{\text{NLOS}}$$

where $\gamma_{\text{NLOS}}$ represents the signal propagation path-loss constant that determines the rate of power loss with increasing distance, $S_{\text{NLOS}}$ refers to the shadow fading parameter, $d$ represents the distance between the transmitting and receiving nodes, $L_0$ represents the RSS at a 1 m distance. Then, for a given $\gamma_{\text{NLOS}}$ and $L_0$, the maximum likelihood (ML) estimate of $d$ can be obtained from the measured RSS index (RSSI) as follows [5,22]:

$$d_{\text{MLE}} = \frac{\text{RSSI}}{10 \gamma_{\text{NLOS}}}$$

Typically, $\gamma_{\text{NLOS}}$ ranges between 1 and 6 (distance model). When the distance between the transmitting and receiving nodes is less than 8 m, the radio propagation model is usually simplified as follows [23]:

$$d = 10^{\frac{\text{RSSI}-40.2}{40.2}} \quad d < 8 \textrm{ m}$$

In fact, it is very difficult to calculate accurate distances between transmitting and receiving nodes under NLOS and multipath conditions. Here, it becomes necessary to empirically characterize the radio propagation model based on a site survey. However, to save the time and labor involved with a site survey, we ask an interesting question: given some unlabeled samples (calibration data) in a time series, how do we choose $\gamma_{\text{NLOS}}$ and $L_0$ that make the distance model best match the actual observations. The specific problem that we wish to solve in this paper is unsupervised learning of the distance model, which is computed by $\gamma_{\text{NLOS}}$ and $L_0$. We address this question with a learning and tracking algorithm based on unsupervised learning.

### 4. PROPOSED ALGORITHM

Next, we introduce the proposed IWLUL algorithm. The basis of the IWLUL algorithm is a specially designed HB-HMM, where the problem for unsupervised learning is formulated in terms of estimating hidden states from the data. Bayesian estimation is conducted via Gibbs sampling, and hidden state estimation is performed via PF. We then discuss our performance analysis of the resulting radio propagation parameters estimation in terms of a derived Posterior Cramér–Rao Lower Bound (PCRLB).

#### 4.1. Hierarchical Bayesian hidden Markov models

For indoor localization, a finite state-space HB-HMM is investigated. Here, a Markov time sequence variable $X_i (i = 1...N)$ takes a value within the finite grid set $\mathcal{X} = \{1,...,K\}$ such that

$$Pr(X_i = k) = \tau(k) \quad \text{and} \quad Pr(X_i = k | X_{i-1} = l) = \phi(k|l)$$

where $\tau(k)$ represents a probability density function of initial state $k$, and $\phi(k|l)$ denotes the probability density associated with a transition from state $l$ to $k$. $\{X_i\}_{i=1...N}$
is modeled using a Markov chain. We focus on estimating \( \{X_i\}_{i=1}^{N} \) from the observation sequence \( \{Y_i\}_{i=1}^{N} \). We assume that the observations \( \{Y_i\}_{i=1}^{N} \) are statistically independent, and their marginal densities are given by

\[
Pr(Y_i = y_i | X_i = x_i) = v(y_i | x_i)
\]  

where \( v(y_i | x_i) \) represents a probability density function of observation \( y_i \) conditioned by \( x_i \). A finite state-space HMM can be defined according to Equations (5) and (6) [24], where Equation (5) defines the evolution of the state with respect to time and Equation (6) defines the likelihood function.

In our approach based on unsupervised learning, the problem is formulated in terms of estimating hidden states from the data. The probability of a location is conditioned by the location that precedes it, and the probability of a given vector \( S_i \) is conditioned by its location. For the \( i \)th location, the states \( \{X_i\} \) correspond to the locations, and \( \{Y_i\} \) correspond to the RSS vector \( \{S_i\} \) that is received each time a state is visited. Specially, rather than estimating a single set of parameters by applying the Baum–Welch expectation maximization algorithm, the HB-HMM approach integrates over all possible parameter values.

For an HB-HMM with parameters \( \theta \), we infer the parameters using the sum rule of probability \( P(B | C) = \int P(B \mid A, C)P(A | C)\,dA \) to integrate out the unknown parameter \( \theta [25]:

\[
P(X | \beta) = \int P(X | \theta, \beta)P(\theta | \beta)d\theta
\]

Because the HB-HMM functions well with the Markov chain Monte Carlo (MCMC) algorithm and PF, it can be used to design a very efficient data fusion algorithm for indoor localization. In this case, the HB-HMM is comprised of two levels. At the higher level, we assume that the multinomial distribution can be parameterized by a single hyper-parameter \( \beta \). At the lower level, we assume that the probabilities of achieving a sensor node location are governed by an HMM in which the distributions over variables are multinomial distributions.

Let probability \( Q = (Q_1, Q_2, ..., Q_k) \) \( Q_1 = \sum_{k=1}^{K} Q_{\psi(l)}|k| \) where \( Q \) can be interpreted as the mixing proportions for the transition probability from the last hidden state \( k \) into a specific hidden state \( l \). Sample states representative of a Markov chain \( X_1, ..., X_N \) are independently drawn from a discrete indicator variable that can assume values with proportions given by \( Q \). The joint distribution of these indicators is multinomial [26]:

\[
P(X_1, ..., X_N | Q) = \prod_{k=1}^{K} Q_k^{n_k}
\]

where \( n_k \) represents the number of times that \( X_i = k \). Because the Dirichlet distribution is a conjugate prior for the multinomial distribution, we parameterize the probabilities \( Q \) by a symmetric Dirichlet process prior with positive concentration hyper-parameter \( \beta \) [27]:

\[
P(Q | \beta) \sim \text{Dirichlet}(\beta / K, ..., \beta / K)
\]

\[
= \frac{\Gamma(\beta)}{\Gamma(\beta / K)^K} \prod_{k=1}^{K} Q_k^{\beta / K - 1}
\]

where \( \Gamma(\cdot) \) represents a gamma function. The parameter \( \beta \) measures the sharpness of the distribution. When \( \beta \geq 1 \), high probability is assigned to uniform distribution, and, when \( \beta < 1 \), multinomial distributions that are closer to sparse multinomials are preferred. According to Equation (7), we can integrate out the probabilities \( Q \) under the hyper-parameter \( \beta \). To do so, we make an exchange of variables to rewrite the integral, \( \int P(X_1, ..., X_N | Q)P(Q | \beta)\,dQ \) into a more readily treatable form. To this end, we adapted the formulae to the \((K-1)\)-dimensional spherical coordinates \((r, \phi_1, ..., \phi_{K-2})\) as follows:

\[
Q_1 = r \cos^2(\phi_1)
\]

\[
Q_2 = r \sin^2(\phi_1) \cos^2(\phi_2)
\]

\[
...
\]

\[
Q_{K-3} = r \sin^2(\phi_1) ... \sin^2(\phi_{K-2}) \cos^2(\phi_{K-2})
\]

\[
Q_{K-2} = r \sin^2(\phi_1) ... \sin^2(\phi_{K-2}) \sin^2(\phi_{K-2})
\]

The integral \( \int P(X_1, ..., X_N | Q)P(Q | \beta)\,dQ \) can be described in terms of a simpler gamma function.

\[
\int P(X_1, ..., X_N | Q)P(Q | \beta)\,dQ = \int P(X_1, ..., X_N | Q)\,dQ
\]

\[
= \int_{0}^{\pi/2} \int_{0}^{\pi/2} ... \int_{0}^{1} \sin^{2(l_1+...+l_{K-2})} + 2(K-2)-1(\phi_1)
\]

\[
\cos^{2l_1+1}(\phi_1) \sin^{2(l_1+...+l_{K-2}) + 2(K-3)-1}(\phi_2)
\]

\[
\cos^{2l_1+1}(\phi_2) ... \sin^{2l_1+1}(\phi_{K-2})
\]

\[
\cos^{2l_1+1}(\phi_{K-2})\

\]

\[
dr d\phi_1 ... d\phi_{K-2}
\]

\[
= \frac{\Gamma(\beta)}{\Gamma(n + \beta)} \prod_{k=1}^{K} \Gamma(n_k + \beta / K) \Gamma(\beta / K)
\]

\[
= \frac{\Gamma(\beta)}{\Gamma(n + \beta)} \prod_{k=1}^{K} \frac{\Gamma(n_k + \beta / K)}{\Gamma(\beta / K)}
\]

This provides the following final expression for the integral:

\[
P(X_1, ..., X_N | \beta) = \int P(X_1, ..., X_N | Q)P(Q | \beta)\,dQ
\]

\[
= \int \frac{\Gamma(\beta)}{\Gamma(\beta / K)^K} \prod_{k=1}^{K} Q_k^{\beta / K - 1}\,dQ
\]

\[
= \frac{\Gamma(\beta)}{\Gamma(n + \beta)} \prod_{k=1}^{K} \frac{\Gamma(n_k + \beta / K)}{\Gamma(\beta / K)}
\]
We use Equation (10)-(12) to integrate out the unknown parameter \( Q \). To facilitate training of the HB-HMM, we must establish the probability that \( X_i \) takes value \( k \) given all the others. Here, we let \( X_{-i} \) represent all the sequence variables except \( X_i \), and \( n_{-ik} \) represent the number of observations, excluding \( X_i \), that take value \( k \). Then the probability \( P(X_i = k | X_{-i}, \beta) \) is easily obtained from (12) (see Appendix A) as follows:

\[
P(X_i = k | X_{-i}, \beta) = \frac{n_{-ik} + \beta / K}{N - 1 + \beta} \quad (13)
\]

For indoor localization, we calculate the likelihood function \( P(Y_j | X_i, X_{-i}) \) using a set of samples based on kernels [28]. The most common kernel is the standard Gaussian kernel. We can approximate the probability density function of \( P(s_{ij} | X_i = k) \) using a Gaussian kernel function as follows by letting \( D_{ij} \) denote the distance from the center of the \( k \)th grid to the \( j \)th reference node, \( \delta(a, b) \) be the Kronecker-delta function (i.e., gives a value 1 if \( a = b \), and 0 otherwise), \( E_{k,j} \) be the expected RSS value of state \( k \) from \( j \)th reference node, \( n_k \) be the number of times that \( X_j = k \), \( \overline{X_j} \) be the expected measurement information (the average RSS measurement from \( j \)th reference node in state \( k \)), \( h \) be the bandwidth which is set as 15 according to empirical analysis.

\[
E_{k,j} = L_0 + \gamma \text{NLOS} \log 10(D_{ij}) + \frac{1}{n_k} \sum_{j=1}^{N} s_{ij} \delta(X_i, k) \quad (14)
\]

\[
P(s_{ij} | X_i = k) = \frac{1}{\sqrt{2\pi n_k h}} \exp \left( -\frac{(s_{ij} - \overline{X_j} - E_{k,j})^2}{2h^2} \right) \delta(Y_i, k) 
\]

We use Equation (14) in the initial training iterations. Considering the high computing complexity of Equation (14), we employ the following form for the remaining iterations:

\[
P(s_{ij} | X_i = k) = \frac{1}{\sqrt{2\pi n_k h}} \exp \left( -\frac{(s_{ij} - \overline{X_j})^2}{2h^2} \right) \quad (15)
\]

Suppose \( Y_j \) is independent from \( X_{-i} \) or \( \beta \), and \( X_i \) is independent from \( S_{-i} \), then

\[
P(Y_j | X_i, X_{-i}) = P(S_j | X_i = k) = \prod_{j=1}^{M} P(s_{ij} | X_i = k) \quad (16)
\]

Next, we can produce the hidden state from the posterior distribution

\[
P(X_i | X_{-i}, S, \beta) \propto P(X_i | X_{-i}, \beta) P(S_i | X_i, X_{-i}) \quad (17)
\]

### 4.2. Bayesian estimation via Gibbs sampling

For the HB-HMM inference problem, we use Gibbs sampling, which is a MCMC technique [25, 29]. Gibbs sampling is ideally suited for inference in graphical models. Gibbs sampling produces state sequences \( X \) sampled from the posterior distribution given by Equation (17).

We initialize a hidden state sequence at random, then iteratively resample each hidden state according to its conditional distribution given the values of all other hidden states and the likelihood probability of the current state. We treat the Gibbs sampling procedure as a gradient search step with the goal of maximizing an evaluation function of the hidden state sequence. In this algorithm, Gibbs Sampling provides a good maximum a posteriori (MAP) estimate. Generally, relative RSS measurements are independent of all but the current state of the node. Thus, the evaluation function for the gradient search step at the \( t \)th iteration is given as

\[
e_t = P(Y_1:n | X_1:n) = \prod_{i=1}^{N} P(S_i | X_i) \quad (18)
\]

From an abstract point of view, the optimization algorithm can be thought as a k-means clustering method constrained by the Dirichlet distribution in dynamic time warping of undivided successive data. Having generated a hidden state sequence, we can facilitate use of a PF method by building the transition matrices of an HB-HMM as

\[
P(X_i = k | X_{i-1} = l, \beta) = \frac{n_{lk} + \beta / K}{\sum_{q=1}^{N-1} n_{lq} + \beta} \quad (19)
\]

where \( n_{lq} = \sum_{i=1}^{N} \delta(X_i, l) \delta(X_{i+1}, q) \)

### 4.3. Hidden state estimation via a particle filter

The Gibbs sampling employed in our optimization algorithm is not ideally suited for frequent updating because the time cost during iterations is substantial (many hours). Moreover, marginalizing over the parameters in any HMM causes long-range dependencies between states [25]. Therefore, for frequent updating, we consider a PF that estimates the sequence of hidden states based on all measurements collected up to the time instant \( N \). A PF is one of the three well-known nonlinear filters, where the other two are the extended Kalman filter and the unscented Kalman filter. A PF often aims to estimate a Bayesian model and is the sequential analogue of MCMC batch methods. Moreover, the method is often much faster than existing MCMC [28]. Based on the results of fingerprint training, use of a PF helps to avoid measurement errors caused by NLOS and multipath conditions.

Generally, a PF estimates \( X_k \) from the proposal distribution \( p(X_k | X_{0:k}, Y_{0:k}) \). However, rather than computing the
4.4. Radio propagation parameters estimation

For the case of updating radio propagation parameters, we derive a ML estimation to jointly compute the path-loss exponent \( \gamma_{\text{NLOS}} \) and the RSS power at 1-m distance \( L_0 \), based on the SDP.

Here, we collected the unknown position data. Then, the range-free algorithm (unsupervised learning algorithm) takes the grid center as the node position. Our goal is for the results estimated by the range-based algorithm to reside closer to the results estimated by the range-free algorithm. We let \( \overline{D}_{kj} \) be the expected mean distance from a mobile node in state \( k \) to the \( j \)th reference node. This is computed by \( \overline{D}_{kj} \), \( \gamma_{\text{NLOS}} \), and \( L_0 \). If we assume that the grid center is the true position of state \( k \), \( \overline{D}_{kj} \) should be close to \( D_{kj} \).

Then, we have the following:

\[
\overline{D}_{kj} = D_{kj} + o_{kj}, \quad o_{kj} \sim N(0, \sigma_{kj})
\]

where we assume each \( o_{kj} \) represents a multiplicative noise following a normal distribution \( N(0, \sigma_{kj}) \) (i.e., \( \overline{D}_{kj} = D_{kj}(1 + N(0, \sigma)) \) where \( \sigma \) represents a constant value). Then, we assume \( \sigma_{kj} = D_{kj} \sigma \) where \( \sigma \) is set as 1 for computational convenience. Therefore, \( \sigma_{kj} = D_{kj} \).

Noise interferes with the acquisition of accurate estimates of the distance from a given point to the reference nodes. We attempt to minimize the effect of noise to obtain new values for \( \gamma_{\text{NLOS}} \) and \( L_0 \) using the ML function \( P(\overline{D}_{kj}, D_{kj}, \gamma_{\text{NLOS}}, L_0) \). These multiplicative noise assumption and ML approach have also been employed in other studies[30]. In our case, the ML estimation is given as follows:

\[
\begin{align*}
\arg \max_{\gamma_{\text{NLOS}}, L_0} & \quad P(\overline{D}_{kj}, D_{kj}, \gamma_{\text{NLOS}}, L_0) \\
= & \arg \max_{\gamma_{\text{NLOS}}, L_0} \prod_{k=1}^{K} \prod_{j=1}^{M} \frac{1}{(2\pi)^{\frac{1}{2}} \sigma_{kj}^2} \\
\cdot \exp \left( -\frac{1}{2\sigma_{kj}^2} (\overline{D}_{kj} - D_{kj})^2 \right) \\
= & \arg \max_{\gamma_{\text{NLOS}}, L_0} \prod_{k=1}^{K} \prod_{j=1}^{M} \frac{1}{(2\pi)^{\frac{1}{2}} \sigma_{kj}^2} \\
\cdot \exp \left( -\frac{1}{2\sigma_{kj}^2} \left( 10^{\frac{-\gamma_{\text{NLOS}}}{10}} - D_{kj} \right)^2 \right)
\end{align*}
\]

Hence, with \( 1 \leq \gamma_{\text{NLOS}} \leq 6 \) and \( 30 \leq L_0 \leq 60 \), the following optimization problem solves the ML estimation problem:

\[
\begin{align*}
\min & \sum_{k=1}^{K} \sum_{j=1}^{M} \frac{1}{\sigma_{kj}^2} \xi_{kj} \\
\text{s.t.} & \\
& \left| 10^{\frac{-\gamma_{\text{NLOS}}}{10}} - D_{kj} \right| \leq \xi_{kj} \\
& \text{for } k = 1..K, j = 1..M \\
& 1 \leq \gamma_{\text{NLOS}} \leq 6 \\
& 30 \leq L_0 \leq 60
\end{align*}
\]

Because the ML estimator (22), which contains \( \phi(x, y) = 10^{\frac{-\gamma_{\text{NLOS}}}{10}} \), is evidently non-convex, finding the global minimum solution of problem (22) is difficult. To provide a convex estimator, other authors have proposed unscented transformation or the Chebyshev norm [8,31].

In the present work, the linearization is obtained through the well-known Taylor series expansion around \( \phi(x_0 = 40, y_0 = 2) \) given by Equation (4). Hence,

\[
\overline{D}_{kj} \approx \phi(x_0, y_0) + \frac{\partial \phi}{\partial x}(x_0, y_0)(x - x_0) + \frac{\partial \phi}{\partial y}(x_0, y_0)(y - y_0)
\]

Then, the ML estimator (22) is redefined as the following:

\[
\begin{align*}
\min & \tilde{b}^T \tilde{X} \\
\text{s.t.} & \\
& \left| \tilde{A}_{kj}^T \tilde{X} + \tilde{c}_{kj} \right| \leq \tilde{b}_{kj}^T \tilde{X} \\
& \text{for } k = 1..K, j = 1..M \\
& \tilde{b}^T \tilde{X} + f \geq 0
\end{align*}
\]

Here, the following definitions apply.

\[
\tilde{b} = \begin{bmatrix}
\frac{1}{\sigma_{1,1}} \\
\vdots \\
\frac{1}{\sigma_{k,j}} \\
\vdots \\
\frac{1}{\sigma_{K,M}}
\end{bmatrix}, \quad \tilde{X} = \begin{bmatrix}
\xi_{1,1} \\
\vdots \\
\xi_{K,M} \\
\gamma_{\text{NLOS}} \\
L_0
\end{bmatrix}, \quad \tilde{b}_{kj} = \begin{bmatrix}
0 \\
\vdots \\
1 \\
0
\end{bmatrix}
\]

\[
\tilde{c}_{kj} = \phi(x_0, y_0) + \frac{\partial \phi}{\partial x}(x_0, y_0)x_0 + \frac{\partial \phi}{\partial y}(x_0, y_0)y_0 - D_{kj}
\]

\[
\tilde{A}_{kj} = \begin{bmatrix}
0 & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & 0 \\
-\log(10) \cdot 10^{(\frac{\sigma_{1,1}}{10})} \cdot (\gamma_{\text{NLOS}} - 40.2) / 400 \\
\vdots & \ddots & \vdots \\
-\log(10) \cdot 10^{(\frac{\sigma_{K,M}}{10})} \cdot (\gamma_{\text{NLOS}} - 40.2) / 20
\end{bmatrix}
\]
The problem (24) is a second-order cone programming problem. The quadratic model can be formulated as follows.

\[
\begin{align*}
\min & \quad \bar{b}^T \bar{X} \\
\text{s.t.} & \quad (\bar{b}_{k,j}^T \bar{X} + c_{k,j}) \geq 0 \\
& \quad k = 1..K, j = 1..M \\
& \quad \bar{b}^T \bar{X} + f \geq 0
\end{align*}
\]

This model represents an SDP problem for which a number of methods and tools are available for obtaining a solution [32].

### 4.5. Proposed algorithm

The IWLUL algorithm is summarized in Algorithm 1. After applying Algorithm 1, we can perform localization by any range-based algorithm.

Here, our goal is to provide propagation model parameters with increased accuracy from a set of unlabeled time series data (calibration data). As such, it is a calibration-effort reduction problem addressed by an unsupervised learning approach. The problem is essentially a regression problem. While Gibbs sampling, PF, kernel density estimation, HMM, and SDP are sophisticated methods, the unsupervised learning approach for indoor wireless localization derived from these methods is not specifically designed for sophistication, but as an effective unsupervised learning approach. Here, we employ a Dirichlet prior and HMM to process the transition probability hidden in the time series. The kernel method is employed to provide clustering of greater density. The kernel method is incorporated into the HMM model to provide the emission probability. So, we think that the Dirichlet prior, transition probability, and high-density clustering contribute greatly to the overall accuracy. Gibbs sampling is not suitable for online tracking, so we use PF. SDP provides global optimization in our regression step. As such, each method tends to contribute to the overall accuracy of the proposed approach and have not been implemented because they are sophisticated, but, rather, because they are effective in the present context.

In our algorithm, the area of the grid affects the results. Large grids result in coarse accuracy, whereas smaller grids cause a larger error rate. In our case, each grid is about 2 x 2 m. Individual cells of the grid that contain a wall or obstruction do not affect the range-free results but do affect the range-based results. Our algorithm seeks to fit the range-based results to the range-free results to mitigate NLOS and multipath conditions. However, as the range-based algorithm provides initial localizations for optimization in our algorithm, the wall or obstruction slightly affects the results.

Addressing NLOS conditions in WSNs is one of the major concerns addressed by our indoor localization system. In our algorithm, we generate a probability model of RSS values for a given location based on the HB-HMM. Next, a radio propagation model is constructed based on SDP. Lastly, mobile locations are generated by a range-based algorithm. Because we directly map out RSS patterns and translate them into distance, measurement errors caused by NLOS are then avoided. Moreover, an unsupervised learning mechanism, which is based on HB-HMM, is used to automatically collect RSS values. Our algorithm offers the most efficient means to construct a training database, saving labor and time because only a single hyper-parameter \( \beta \) is required in the construction phase for the HB-HMM.

On the other hand, our indoor localization estimation can accumulate information from consecutive measurements because we utilize a transition model for time series analysis.

In our case, collecting a large amount of training data is not difficult because of the unsupervised learning algorithm employed. With decision stumps, all learning algorithms achieve higher accuracies with a larger dataset. This makes it possible for our system to locate mobile devices in an indoor environment more accurately.

![Algorithm 1 IWLUL: Indoor Wireless Localization based on Unsupervised Learning.](image)

**Algorithm 1 IWLUL: Indoor Wireless Localization based on Unsupervised Learning.**

**Require:** Collected RSS database and hyper-parameter \( \beta \);

**Ensure:** estimated propagation parameters \( \gamma^{\text{NLOS}}, L_0 \);

1. Initialize parameters \( \gamma^{\text{NLOS}} = 2, L_0 = 40.2 \);
2. Repeat
3. Instantiate a random hidden state sequence;
4. Repeat
5. Compute count matrices of \( n_k, n_{i,k} \);
6. Gibbs sampling by applying Equation (17);
7. Compute evaluation function \( e_t \);
8. If \( e_t \geq e_{t-1} \) then
9. The acceptance probability = 1.
10. Else
11. The acceptance probability = \( \exp(-4(e_t - e_{t-1})/e_t) \).
12. End if
13. If any \( n_k \leq N \) then
14. The acceptance probability = 0.
15. End if
16. Until convergence or iteration limit reached
17. Estimate hidden states by Particle Filter.
19. Until convergence or iteration limit reached
20. Return \( \gamma^{\text{NLOS}}, L_0 \);
4.6. Performance analysis by posterior Cramér-Rao lower bound

In this final subsection, we use the PCRLB as a metric for tracking performance. The PCRLB presents the smallest quadratic error that an algorithm can theoretically achieve. In accounting for the measurement model and motion model, the PCRLB is meaningful for Bayesian inference methods.

In most instances, the following linear motion model can be employed:

$$\theta_{t+1} = F_t \theta_t + W_t \quad F_t = \text{unit matrix}$$

where $F_t$ represents a unit matrix that models the position dynamics, the mobile node’s position at time $t'$ is defined as $\theta_{t'} = [v_{t'}, v_{t'}^T]$, and $W_t$ follows the normal distribution $N(0, Q_t)$. This motion model is suitable for many commercial applications such as an indoor robot cleaner and shopping mall monitoring. Regarding Equation (2), the measurement is modeled by

$$S_{t'} = H(\theta_{t'}) + V_{t'}$$

$$d = [\|\theta_{t'} - r_1 \|, \ldots, \|\theta_{t'} - r_{\alpha} \|]$$

$$H(\theta_{t'}) = L_0 + 10 \eta_{NLOS} \log_{10} (d)$$

where $S_{t'}$ represents the fingerprint vector at time $t'$, $r_j$ represents the position of the $j$th reference node and $V_{t'} \sim N(0, R_{t'})$, $R_{t'} = diag(\sigma_0^2, \sigma_0^2)$. Then, the PCRLB ($J_{t'}^{-1}$) for estimating $\theta_{t'}$ using $S_{t'}$ is given by

$$E \left[ \left( \hat{\theta}_{t'} - \theta_{t'} \right) \left( \hat{\theta}_{t'} - \theta_{t'} \right)^T \right] \geq J_{t'}^{-1}$$

where $J_{t'}$ represents the posterior Fisher information submatrices for estimating state vectors $\theta_{t'}$ and $\hat{\theta}$ represents an unbiased estimator of $\theta$.

The Tichavsky method and the sequential Monte Carlo method have been proposed to calculate $J_{t'}$[33]. Because the Tichavsky method is relatively easy to apply, we adopt it here to approximate a lower bound on the mean square error of our tracking method. The Tichavsky method calculates $J_{t'}$ recursively without the need to invert large matrices [34]. Based on Fisher information submatrices $J_{t'}$, the method provides

$$J_{t'+1} = D_{t'}^{22} - D_{t'}^{21} \left( J_{t'} + D_{t'}^{11} \right)^{-1} D_{t'}^{12}$$

where

$$\frac{\partial H}{\partial X_{t'}} = \frac{10 \eta_{NLOS}}{\ln 10}$$

$$J_{t'+1} = J_X + J_Z$$

$$J_X = \left( \hat{Q}_{t'} + J_{t'}^{-1} \right)^{-1}$$

$$J_Z = \frac{\partial H^T R_{t'}^{-1} \partial H}{\partial X_{t'}}$$

The range-based algorithm contribution is given by $J_Z$ [36]. The transition model contribution is then given by $J_X$. In this case, it is evident that the proposed algorithm theoretically outperforms strictly range-based algorithms.

5. EXPERIMENTAL RESULTS

The proposed algorithm can be applied to numerous indoor wireless network problems. In this section, we demonstrate the effectiveness and accuracy of this unsupervised learning algorithm in an actual indoor environment comprised of a single room with a single mobile device.

5.1. Room plan

In this selected scenario, an IEEE 802.15.4 wireless network is deployed in a rectangular-shaped room covering an area of 4.5 $\times$ 7 m. As shown in Figure 1, a coffee table is positioned in the center of the room with three sofas positioned around. The room is divided into six grids ranging from 1 to 6, as marked by encircled numbers within the figure. Four reference nodes are marked on the floor plan using small-filled circles. A total of six nodes are employed, including the four reference nodes, a blind node, and a gateway. In our indoor localization problem, each state is represented by a room grid.

Figure 1. Room plan of our experiments.
5.2. Experimental platform

The system platform is built upon the Texas Instruments Z-Stack\textsuperscript{TM} ZigBee\textsuperscript{®} protocol stack. The reference node uses an IEEE 802.15.4-compliant 2.4 GHz Chipcon CC2430 radio. The blind node is equipped with a Chipcon CC2431 radio. The WLAN interface uses Chipcon CC2430. The positioning system consists of a host computer, four reference nodes, a blind node, and a gateway node.

5.3. Experiments and results analysis

The mobile node is programmed to move around the room at a velocity of 1 m/s. Every 0.3 s, the mobile node receives four RSS values from the four reference nodes. The total time for a single experimental sample is about 7 min. In total, we collected 16 samples, 10 for training, and 6 with known mobile node positions for testing. Using the IWLUL algorithm, we obtained the HB-HMM and the parameters of the distance model for indoor localization.

The accuracy rate is the most important factor of a localization system. In our experiment, the best accuracy rate of our localization algorithm was 81.53%. The results indicate that accuracy is best when $\beta \leq 3000$. This can be observed in Table I. When $\beta$ is larger, the distribution of the state variable is more uniform. Figure 2 directly shows the location distributions for a single trial. It is also suggested from Figure 2 that the position distributions in the grid follow Gaussian distributions as expected.

In addition to the accuracy, an analysis of location error should also consider the distribution of the location error between the estimated position and the true position. A cumulative distribution function (CDF) is often utilized to measure the distribution of location error. For comparison with the proposed algorithm, a typical range-based algorithm based on WLS [37], the HIWL algorithm based on unsupervised learning [17], and the EasyLoc algorithm [21] were also employed within the experimental platform, and the CDF was collected for each. Here, the WLS algorithm is a range-based method, the HIWL algorithm is a range-free method, and the EasyLoc algorithm used both range-based and range-free localization. When $\beta = 1800$, the proposed algorithm demonstrated a location precision of 81.53% within 1.5 m (the CDF of the location error within 1 m was 0.6945), 86.17% within 2 m, 94.56% within 3 m, and 99% within 4 m. As shown in Figure 3,

<table>
<thead>
<tr>
<th>Value of $\beta$</th>
<th>$L_0$</th>
<th>$\gamma_{\text{NLOS}}$</th>
<th>$\log(e_1)$</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>52.5</td>
<td>2.21</td>
<td>34.1083</td>
<td>62.30%</td>
</tr>
<tr>
<td>6.0</td>
<td>53.8</td>
<td>2.20</td>
<td>30.0451</td>
<td>51.20%</td>
</tr>
<tr>
<td>30.0</td>
<td>56.9</td>
<td>1.71</td>
<td>29.7629</td>
<td>35.80%</td>
</tr>
<tr>
<td>600</td>
<td>51.2</td>
<td>2.22</td>
<td>177.527</td>
<td>79.73%</td>
</tr>
<tr>
<td>1800</td>
<td>52.7</td>
<td>2.71</td>
<td>189.765</td>
<td>81.53%</td>
</tr>
<tr>
<td>2100</td>
<td>51.2</td>
<td>3.08</td>
<td>178.551</td>
<td>80.81%</td>
</tr>
<tr>
<td>3000</td>
<td>52.4</td>
<td>2.88</td>
<td>186.0226</td>
<td>80.1%</td>
</tr>
<tr>
<td>6000</td>
<td>52.5</td>
<td>2.64</td>
<td>177.5211</td>
<td>76.7%</td>
</tr>
<tr>
<td>30000</td>
<td>52.4</td>
<td>2.65</td>
<td>174.5515</td>
<td>72.5%</td>
</tr>
</tbody>
</table>

Table I. The HB-HMM with different hyper-parameter values.

HB-HMM, hierarchical Bayesian hidden Markov model; NLOS, non-line-of-sight.
Figure 4. Performance in comparison with root mean squared error (RMSE).

the performance of the IWLUL algorithm is better than all others considered.

The experimental results of the algorithms considered are also compared with the PCRLB of our algorithm. To compute the PCRLB of the algorithm, we set the following matrices:

\[
J_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \hat{Q}_k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \hat{R}_k = \begin{bmatrix} 1.8 & 0 & 0 & 0 \\ 0 & 1.8 & 0 & 0 \\ 0 & 0 & 1.8 & 0 \\ 0 & 0 & 0 & 1.8 \end{bmatrix}
\]

The results of comparison using the root mean squared error as a function of sampling time (given in increments of 0.3 s) are shown in Figure 4. In this test, the proposed algorithm attains an accuracy of roughly 1.2 m relative to 3 m for the other range-based algorithms considered (see the presented WLS data given in Figure 4. and [22]) and it results in a remarkable improvement over HIWL algorithm and Easyloc algorithm. The figure also shows that typical range-based positioning systems perform poorly using simplified models.

In our experiments, if we had placed the reference nodes on the ceiling or some other position other than the floor, the relative performance of the proposed approach would have been more convincing. This is due to the fact that at the floor level, the signal suffers from shadowing [3]. However, we placed the four reference nodes on the floor, which is indicative of actual NLOS conditions, and, under equivalent conditions, our algorithm was objectively demonstrated to outperform several related algorithms. Considering that our algorithm did not increase the labor costs beyond the costs required by the algorithm in [21], the performance of our algorithm is significantly better. In [38], Correa et al. have developed an enhanced filtering method for indoor positioning and tracking applications using a WSN. They take into account the inertial measurements and RSS measurements. They have tested their algorithm in a scenario which was a 5.6 × 13.6 m room covered with eight 802.15.4 reference nodes. The scenario was similar to our scenario which was a 4.5 × 7 m room covered with four 802.15.4 reference nodes. Their algorithm exhibited a localization precision of 80% within 1.5 m. The performance was similar to ours but we did not use the inertial measurements. In [3], the area of the experimental site was greater than the area employed in our experiments, the RADAR algorithm [39] (a classic fingerprinting algorithm) with ten 802.15.4 reference nodes in a 219 × 169 ft site exhibited a localization precision of 80% within 30 ft. However, our algorithm demonstrated a precision of 80% within 1.5 m (4.8 ft). As such, the results appear to be rather convincing. However, because the Correa’s algorithm employs the inertial measurements and the RADAR algorithm employs labeled data, they could not be included in our experiments.

The computational complexity of a localization system is a significant factor because of the hardware, software, and operational requirements of a system. The experimental tests verified that the computational complexity of the proposed algorithm is equivalent to those of the other range-based algorithms.

6. CONCLUSION

To solve the indoor localization problem for WSNs, an unsupervised algorithm has been proposed to train a range-based algorithm offline. The range-based algorithm also provides good initial localizations for global optimization in unsupervised learning. We train the model using HB-HMM and PF, and use the range-based algorithm online. The proposed algorithm presents four main benefits. First, unlike conventional range-free algorithms, a site survey is no longer required. Second, rather than employing random initial parameters for the HMM, we use a single hyper-parameter for global optimization. Third, the grid-based algorithm can utilize conventional range-based methods to directly convert RSS to distance. Fourth, because WSNs are already equipped with the necessary RSS units for communication purposes, our localization estimator can be layered over this existing capability with low computational cost. Extensive experiments were conducted to evaluate our proposed algorithm. Experiment results verify that the proposed algorithm outperforms other range-based algorithms without adding computation cost.

APPENDIX A: PROOF OF THE CONDITIONAL PRIOR FOR GIBBS SAMPLING

We note the following.

\[
n_k = \begin{cases} 
n_{i,k} & \text{if } X_i \neq k \\ 
n_{i,k} + 1 & \text{if } X_i = k 
\end{cases}
\]
\[ P(X_i = k | X_{-i}, \beta) \propto P(X_i = k, X_{-i}) \beta \]
\[ = \frac{\Gamma(\beta)}{\Gamma(n + \beta)} \prod_{k=1}^{K} \frac{\Gamma(n_k + \beta / K)}{\Gamma(\beta / K)} \]
\[ \propto \prod_{k=1}^{K} \Gamma(n_k + \beta / K) = \Gamma(n_k + \beta / K) \prod_{l \neq k} \Gamma(n_{-l} + \beta / K) \]
\[ = \Gamma(n_{-i,k} + 1 + \beta / K) \prod_{l \neq k} \Gamma(n_{-i,l} + \beta / K) \]
\[ = (n_{-i,k} + \beta / K) \prod_{l \neq k} \Gamma(n_{-i,k} + \beta / K) \]
\[ \propto (n_{-i,k} + \beta / K) \quad (A2) \]

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**AUTHORS’ BIOGRAPHIES**

**Li Li** received the BS degree in information Engineering from the Xiangtan University, Hunan, China, in 1996 and received the MS degree in information Engineering from Central South University, Changsha, China, in 2005. He is currently working toward the PhD degree with the School of Information and Engineering, Central South University. His current research interests include wireless localization, the application of machine learning, and convex optimization techniques.

**Wang Yang** received his BS degree in Computer Science and Technology from National University of Defense Technology, China, and PhD in Computer Science and Technology from Tsinghua University, China. He is now an assistant professor in School of Information Science and Engineering, Central South University, China. His research interests include mobile computing, networking protocol, and machine learning.
Md Zakirul Alam Bhuiyan received the PhD degree and the MEng degree from Central South University, China, in 2009 and 2013 respectively, all in Computer Science and Technology. He is currently an assistant professor (research) in the Department of Computer and Information Sciences at Temple University. He is a member of the Center for Networked Computing (CNC). Earlier, he worked as a postdoctoral fellow at the Central South University, China, a research assistant at the Hong Kong PolyU, and a software engineer in industries. His research focuses on dependable cyber physical systems, wireless sensor network applications, network security, and sensor-cloud computing. He has served as a managing guest editor, workshop chair, publicity chair, TPC member, and reviewer of international journals/conferences. He is a member of IEEE and a member of ACM.

Guojun Wang received his BS degree in Geophysics, MS in Computer Science, and PhD in Computer Science from Central South University, China. He is currently a Professor of Guangzhou University and Central South University. He has been an Adjunct Professor at Temple University, USA; a Visiting Scholar at Florida Atlantic University, USA; a Visiting Researcher at the University of Aizu, Japan; and a Research Fellow at the Hong Kong Polytechnic University. His research interests include network and information security, Internet of things, and cloud computing. He is a distinguished member of CCF, and a member of IEEE, ACM, and IEICE.